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LE PROBLEME DU PLUS COURT CHEMIN PASSANT PAR UN ENSEMBLE DONNE DE NŒUDS

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Rapport de Recherche

Titre

Le problème du plus court chemin passant par un ensemble donné de noeuds[1]

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Résumé : Considérons un graphe orienté $G = (V, A)$ ayant pour ensemble de noeuds V et pour ensemble d'arcs A. Soit c_{uv} la longueur d'un arc $uv \in A$. Étant donnés deux noeuds distincts s et t de V, nous nous intéressons au problème de déterminer un plus court chemin élémentaire (en longueur) de s à t dans G qui doit visiter une seule fois tous les noeuds d'un ensemble donné $P \subseteq V - \{s, t\}$, mais pas nécessairement que ces noeuds. Ce problème est NP-difficile pour $P = V - \{s, t\}$. Une formulation classique de programmation entière basée sur des flots, en tenant en compte des contraintes de visite des noeuds dans P, donne souvent des solutions relaxées en raison de la possible existence de cycles. Ainsi, nous développons deux formulations compactes pour ce problème. L'une est basée sur une version adaptée des contraintes d'élimination de cycle du polytope des arbres couvrant d'un graphe et l'autre est une nouvelle formulation MIP primal-duale. Des expériences numériques sont très encourageants.
Mots-clés: Plus courts chemin avec contrainte de visite, formulations compacte, formulation primal-duale.

Shortest-paths visiting a given set of nodes

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Abstract

Consider a directed graph $G = (V, A)$ with set of nodes V and set of arcs A and let c_{uv} denote the length of an arc $uv \in A$. Given two distinguished nodes s and t of V we are interested in the problem of determining a shortest-path (in length) from s to t in G that must visit only once all nodes of a given set $P \subseteq V - \{s, t\}$, but not necessarily only these nodes. This problem is NP-hard for $P = V - \{s, t\}$. A classic integer programming flow based formulation for this problem taking into account the visiting constraints for nodes in P gives relaxed solutions due to the possible existence of cycles. Thus, we develop two compact extended formulations for this problem. One is based on an adapted version of the cycle elimination constraints of the spanning tree polytope and the other is a new primal-dual based mixed integer formulation. Numerical experiments are very encouraging.

Keywords: combinatorial optimization, shortest path visiting given nodes, compact extended formulation, linked dual-primal formulation

1. Introduction

The $(s-P-t)$−shortest-path (for short) in a directed graph $G = (V, A)$ with set of nodes V and set of weighted arcs A consists in finding a path of minimum length between an origin node $s \in V$ and a destination node $t \in V$ that visits only once all nodes of a given set $P \subseteq V - \{s, t\}$. We know that for $P = V - \{s, t\}$ the problem is equivalent to find an Hamiltonian path of minimum length in G, which is NP-hard. Surprisingly, in a brief literature review, we find only few works on this problem (Dreyfus (1969); Ibaraki (1973); Saksena and Kumar (1966)).

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It seems that the first (and erroneous) algorithm for this problem is due to Saksena and Kumar (1966). Dreyfus (1969) proposes to solve the problem by reducing it to an instance of the traveling salesman problem. Ibaraki (1973) introduces an exponential dynamic programming algorithm and a branch and bound (B&B) method. Ibaraki's model used in the B&B algorithm is defined only with continuous variables. His model is equivalent to the well known flow formulation of the shortest-path problem, thus relaxed node solutions in the search B&B tree are integer and present at least one cycle when the node solution is not an elementary path. The idea behind the Ibaraki's B&B algorithm is to fix at zero (one at a time) an arc of a given cycle C of a B&B node solution as branching rule to create $|C|$ new B&B subproblems. This means possibly enumerating all cycles of G in a B&B tree, because relaxed solutions of the flow based model in Ibaraki (1973) are weak.

This motivates us to develop new formulations for this problem. We adapt the cycle elimination constraints of the compact extended formulation for the spanning tree polytope of an undirected graph in Conforti et al. (2010) and Yannakakis (1991) to deal with the oriented arcs of the $(s-P-t)-$shortest-path problem. Moreover, we explore a nice property of elementary paths to obtain a primal-dual based mixed integer compact extended formulation. The novelty is to characterize feasible solutions by linking primal and dual variables in a same set of constraints exploring that property. On the best of our knowledge, this is the first work exploring these techniques for solving the $(s-P-t)-$shortest-path problem.

2. Problem formulation

Consider $G=(V,A)$ a directed graph with set of nodes V and set of weighted arcs A. Let $c_{uv}\in\mathbb{R}_+$ represent the length of arc $uv\in A$. The problem is to determine an elementary path in G of minimum length between an origin node $s\in V$ and a destination node $t\in V$ that visits a given set $P\subseteq V-\{s,t\}$. We represent a $(s-P-t)$-path in G by a vector $x\in\{0,1\}^{|A|}$, where $x_{uv}=1$ if uv belongs to the $(s-P-t)$-path, and $x_{uv}=0$, otherwise.

Thus, a mathematical model for this problem is

$$(Q) \quad \min_{x\in\{0,1\}^{|A|}} \quad \sum_{uv\in A} c_{uv}x_{uv} \tag{1}$$

$$s.t. \quad \sum_{i \mid iv\in A} x_{iv} - \sum_{j \mid vj\in A} x_{vj} = \begin{cases} 1, & \text{if } v = s \\ -1, & \text{if } v = t \\ 0, & \text{otherwise} \end{cases} \quad \forall v \in V \tag{2}$$

$$\sum_{u\in V \mid uv\in A} x_{uv} = 1, \ \forall\, v \in P \tag{3}$$

$$\sum_{uv\in A(S)} x_{uv} \leq |S| - 1, \ \forall\, S \subset V \tag{4}$$

where $A(S)$ represents the set of arcs with both extremities in S. Constraints (2) define an unrestricted $(s-t)$-path in G. In (3) we impose that each node $v \in P$ must be visited by imposing that one arc enters v. Constraints (4) avoid the existence of cycles in any solution. Note that the number of these sub-tour elimination constraints is exponential. In this case, one can try to solve problem (Q) iteratively by relaxing the constraints (4) and cutting off cycles obtained at each iteration. This means solving a MIP model each iteration until its corresponding solution presents no cycle. Eliminating cycles is also the idea of the branch-and-bound algorithm in Ibaraki (1973), where the authors use a flow-based model that is equivalent to the one defined by (1)-(3).

Alternatively, we can adapt compact extended formulations for the spanning tree polytope of a non oriented and complete graph in Conforti et al. (2010) and Yannakakis (1991) to deal with non complete digraphs. The sub-tour elimination constraints in these works are obtained based on rooted spanning trees and they avoid cycles as well as the ones in (4).

To introduce our new formulation for the minimum length $(s-P-t)$-path of $G = (V, A)$, we define the set $A^+ := A \cup \{(u, v) \in V \times V \mid (v, u) \in A\} \cup \{(u, v) \in V \times V \mid u \neq v,\ (u, v) \notin A\}$. Note that A^+ contains all possible arcs for G as if it were a complete digraph. This helps to adapt the ideas in Conforti et al. (2010) and Yannakakis (1991) to our problem. Indeed, for any node $k \in V$, let T_k represent the path-arborescence obtained by rooting a given $(s-P-t)$-path of G at k. In this case, T_k is a k-rooted pending path where the root node k in level zero is father of any node j in level one if arc (k, j) or (j, k) belongs to T_k (we do not consider the orientation of the arcs in T_k) and, in general, a node j in level l is son of a node i in level $l-1$ if i is nearer k than j and if (i, j) or (j, i) belongs to T_k. We also define, following the interpretation of Conforti et al. (2010) (for spanning trees), for every $k \in V$ and for all $(u,v) \in A^+$, binary variables $\lambda_{kij} = 1$ if j is father of i in T_k, and $\lambda_{kij} = 0$, otherwise. A compact extended formulation for (Q) is then

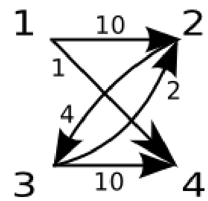
$$
\begin{aligned}
(Q2) \quad \min_{x \in \{0,1\}^{|A^+|}} \quad & \sum_{uv \in A} c_{uv} x_{uv} && (5)\\
s.t. \quad & (2) - (3) && \\
& \lambda_{kij} + \lambda_{kji} \geq x_{ij}, \ \forall\, i,j,k \in V,\ i \neq j && (6)\\
& \textstyle\sum_{j \in V-\{i\}} \lambda_{kij} \leq 1, \ \forall\, i,k \in V,\ i \neq k && (7)\\
& \lambda_{kkj} = 0, \ \forall\, j,k \in V,\ k \neq j && (8)\\
& x_{uv} = 0, \ \forall\, (u,v) \in A^+ - A && (9)\\
& x_{uv} + x_{vu} \leq 1, \ \forall\, (u,v),(v,u) \in A && (10)\\
& \lambda \in \{0,1\}^{|V \times A^+|} && (11)
\end{aligned}
$$

In model $(Q2)$, (2)-(3) establish that there is a path between s and t in G and that the nodes in P are visited. Constraints (6) estate that if an arc (i,j) is in the solution (i.e. $x_{ij} = 1$), then or j is father of i or i is father of j in any k-rooted path T_k of G. Constraints (7) limit to at most one father for any node i in T_k, with $i \neq k$. Constraints (8) estate that none node j can be father of node k in a k-rooted path T_k. Constraints (9) fix at zero all the corresponding variables related to the extra arcs we add to make G a complete digraph. Constraints (10) avoid obtaining a cycle $C = \{(i,j),(j,i)\}$ between any pair of nodes i and j when both these arcs belong to A. The reader should note that a straightforward adaptation for digraphs of the sub-tour elimination constraints in Conforti et al. (2010) (with equalities in (6) and (7)) does not work for this problem. Indeed, if we consider that exactly one of the arcs $(i,j) \in A$ or $(j,i) \in A$ is present in a given solution, then if (6) were written as equality, the resulting model would be infeasible due to the presence of the inconsistent equations $\lambda_{kij} + \lambda_{kji} = 1$ and $\lambda_{kij} + \lambda_{kji} = 0$, for all $k \in V$. Moreover, (7) can be written as equality only if $P = V - \{s,t\}$, meaning that $\lambda_{kij} = 1$, for some (k,i,j)-tuple and, consequently, that $x_{ij} = 1$ or $x_{ji} = 1$ (i.e. imposing the connectivity of every node i). This is the reason why our model follows basically the one in Yannakakis (1991). As we can see in Proposition 1, the constraints (10) are necessary for obtaining cycle-free solutions for the problem.

Proposition 1. *Relaxing constraints* (10) *in the model* $(Q2)$ *possibly leads to the occurrence of cycles in the relaxed problem solution.*

Proof 1. *We show an example where the optimal solution of* (5)-(11)*, without the constraints* (10)*, contains a cycle. Consider the digraph in Figure 1.*

Figure 1: A digraph $G = (\{1,2,3,4\}, \{(1,2),(1,4),(2,3),(3,2),(3,4)\})$. The arc lengths are presented near each arc.



The optimal solution of value 7 *for the resulting model presents* $\bar{x}_{14}$, $\bar{x}_{23}$, $\bar{x}_{32}$ *and* $\bar{\lambda}_{123}$, $\bar{\lambda}_{132}$, $\bar{\lambda}_{141}$, $\bar{\lambda}_{214}$, $\bar{\lambda}_{232}$, $\bar{\lambda}_{241}$, $\bar{\lambda}_{314}$, $\bar{\lambda}_{323}$, $\bar{\lambda}_{341}$, $\bar{\lambda}_{414}$, $\bar{\lambda}_{423}$, $\bar{\lambda}_{434}$*, all equal to* 1*, with all the remaining variables being* 0*. Note, in this case, that the values of the* λ *variables do not correspond to the interpretation we give them. However, when considering* (10)*, the optimal solution of value* 24 *is* $\bar{x}_{12}$, $\bar{x}_{23}$, $\bar{x}_{34}$ *and* $\bar{\lambda}_{121}$, $\bar{\lambda}_{132}$, $\bar{\lambda}_{143}$, $\bar{\lambda}_{212}$, $\bar{\lambda}_{232}$, $\bar{\lambda}_{243}$, $\bar{\lambda}_{312}$, $\bar{\lambda}_{323}$, $\bar{\lambda}_{343}$, $\bar{\lambda}_{412}$, $\bar{\lambda}_{423}$, $\bar{\lambda}_{434}$*, all equal to* 1*, with all the remaining variables being* 0*.* □

Now consider the following trivial property before introducing our second compact extended formulation for the problem.

Property 1. *If* $\{(s,s_1),(s_1,s_2),\cdots,(s_{p-1},s_p),(s_p,t)\}$ *is a minimum length* $(s-P-t)$*-path of* $G=(V,A)$*, with* $P \subseteq \{s_1,s_2,\cdots,s_p\}$ *and* $\pi(v)$ *denotes the distance from node* v *to* s *in this path, for all* $v \in \{s,s_1,s_2,\cdots,s_p,t\}$ *, then* $\pi(s)=0$*,* $\pi(s_1)=c_{s,s_1}$*,* $\pi(s_j)=\pi(s_{j-1})+c_{s_{j-1},s_j}$*, for* $j \in \{2,3,\cdots,p\}$*, and* $\pi(t)=\pi(s_p)+c_{s_p,t}$*.*

We know that the unrestricted version of the minimum length $(s-P-t)$-path problem (i.e. for $P=\emptyset$) can be solved by model (1)-(2). In this case, if we associate dual variables $\pi \in \mathbb{R}^{|V|}$ with the constraints (2), then by duality theory we have that $\pi(v)-\pi(u) \le c_{uv}$, for all $(u,v) \in A$, with $\pi(s)=0$. Note that this inequality is not valid for the $(s-P-t)$-path problem due to the presence of the constraints (3). However, as Property 1 must apply, we need to worry only with the dual multipliers related to the nodes in the solution path.

Therefore, we propose the following approach where we put together in a same model the primal and the dual variables x and π, respectively. Our idea is to characterize feasible solutions by linking primal and dual variables in a same set of constraints in order to satisfy the Property 1 and, consequently, to avoid cycles in any solution.

In the next model consider $\mathcal{M}$ a very large positive constant. The variables are the same as those defined in the above paragraphs.

$$
\begin{aligned}
(Q3) \qquad & \min_{x\in\{0,1\}^{|A|}} \sum_{uv\in A} c_{uv}x_{uv} && (12)\\
s.t. \qquad & (2)-(3) && \\
& \pi(v)-\pi(u) \le c_{uv} + \mathcal{M}(1-x_{uv}),\ \forall\,(u,v)\in A && (13)\\
& \pi(v)-\pi(u) \ge c_{uv} - \mathcal{M}(1-x_{uv}),\ \forall\,(u,v)\in A && (14)\\
& \pi(s)=0, \pi \ge \mathbf{0} && (15)
\end{aligned}
$$

In model $(Q3)$, constraints (13) and (14) impose that if an arc (u,v) is in the solution, then the Property 1 is satisfied because they became an equality constraint $\pi(v)-\pi(u)=c_{uv}$ for this arc; otherwise, both constraints became redundant. Consequently, no cycle can be present in a feasible solution.

3. Computational experiments

We run our instances in a PC Core 2 Duo P8600 (2.4GHz - 4G RAM) using IBM ILOG CPLEX 12.3. The path's origin and destination of all instances are the nodes 1 and $|V|$, respectively. These instances are random generated digraphs with integer arc lengths randomly chosen from the interval $[1,50]$. The set of arcs A is obtained according to a predetermined probability. The cardinality of the set P are given and its elements are chosen randomly. These parameters appear in each instance identifier. We adopt $\mathcal{M}=10000$ in the model $(Q3)$ for all instances.

The legend in Table 1 is as follows. The first column presents the instance identifier $Inst$ composed of three parts $Prob+|V|+|P'|$: the first character indicates the probability $Prob$ used to consider or not the arcs in A (they are represented by letters **a**, **b**, **c** and **d** indicating probabilities $Prob=0.2$, $Prob=0.4$, $Prob=0.7$ and $Prob=1.0$, respectively); the second part indicates the number of nodes $|V|$ of G; and the third part indicates the number of nodes in $|P|\cup\{s,t\}$ (they are also indicated by letters **a**, **b**, **c** and **d** at the end of the instance identifier indicating cardinalities $|P'|=0.25|V|$, $|P'|=0.50|V|$, $|P'|=0.75|V|$ and $|P'|=1.00|V|$, respectively). The value of the continuous relaxed solution and of the optimal solution for each model are denoted by w and z, respectively. The CPU time (in seconds) to obtain the continuous relaxed solution and the optimal solution for these models are denoted by t_r and t, respectively. The total number of CPLEX MIP iterations and CPLEX branch-and-bound nodes to obtain the optimal solution for each model are denoted by mip and bb, respectively.

The first element we compare in the Table 1 is the quality of the linear relaxation of the models $(Q2)$ and $(Q3)$. We reach 16 optimal lower bounds in column w with the model $(Q2)$ (from a total of 48 instances and observing that the arc lengths are integer), while only 5 optimal lower bounds are reached with the model $(Q2)$. In general, linear relaxed solutions obtained with the model $(Q2)$ are larger than those obtained with the model $(Q3)$. The execution times to obtain the linear relaxed solution with the model $(Q2)$ are very large when compared to the ones related to the model $(Q3)$. Observe that the lower bound obtained with the model $(Q2)$ can be considered very close to the optimal integer solution values reported in the column z. This seems to explain why CPLEX spent a high effort in solving the related integer model by the MIP approach and calling the branch-and-bound method only for few instances. The second element of our analysis concerns the quality of the optimal integer solutions. Both models reach all optimal solutions (except for the instance $a20c$ that has no feasible integer solution). The number of CPLEX MIP iterations in the column mip to obtain the optimal integer solution with the model $(Q2)$ is very large when compared to the ones obtained with the model $(Q3)$ (in only one case, for the instance $b80d$, this parameter was larger for the model $(Q3)$). The CPLEX branch-and-bound method is called in 20 and 22 instances, for the models $(Q2)$ and $(Q3)$, respectively. In this occasion, the number of branch-and-bound nodes (in the column bb) in the model $(Q3)$ is larger the the related one in the model $(Q2)$ for 17 instances, being smaller only for 11 instances. The execution times to obtain the integer optimal solution with the model $(Q2)$ are much larger than those obtained with the model $(Q3)$ (except for the instance $c40b$, where the execution time is larger for the model $(Q3)$).

In our experiments there are no conclusive elements to characterize the problem difficulty in terms of the digraph density (given by the probability we use to construct the set of arcs of each instance) and the cardinality of $|P|$. If we observe the execution time t or the number of MIP iterations mip of both models $(Q2)$ and $(Q3)$, there is no expressive concentration of difficult instances in any combination of these parameters.

To conclude our analysis, we observe that the good quality of the linear relaxed solution of the model $(Q2)$ is not sufficient for CPLEX saving execution time in solving these instances.

Table 1: Numerical results for the models ($Q2$) and ($Q3$) by using CPLEX 12.3.

	Model ($Q2$)						Model ($Q3$)					
$Inst$	w	t_r	z	mip	bb	t	w	t_r	z	mip	bb	t
a20a	116.00	0.04	201	8203	62	0.98	89.06	0.00	201	2185	271	0.32
a20b	219.00	0.05	219	266	0	0.15	197.10	0.00	219	43	0	0.02
a20c	410.00	0.14	*	1907	0	0.54	344.19	0.00	*	7	0	0.01
a20d	384.00	0.05	409	1767	0	0.45	359.03	0.00	409	68	0	0.02
b20a	104.00	0.06	114	1246	0	1.25	99.02	0.00	114	311	19	0.45
b20b	148.00	0.07	148	586	0	0.26	148.00	0.00	148	50	0	0.01
b20c	243.33	0.16	249	3880	3	8.07	242.01	0.01	249	62	0	0.02
b20d	157.67	0.27	171	4506	0	4.81	144.01	0.02	171	1948	217	0.27
c20a	53.00	0.06	57	386	0	0.50	49.00	0.01	57	62	0	0.07
c20b	77.00	0.12	77	530	0	0.37	77.00	0.00	77	41	0	0.03
c20c	101.00	0.20	101	1055	0	0.67	99.00	0.00	101	69	0	0.05
c20d	113.00	0.30	113	1280	0	0.41	106.00	0.01	113	119	0	0.08
d20a	25.50	0.08	26	334	0	0.56	25.00	0.02	26	35	0	0.06
d20b	52.42	0.22	56	2091	0	2.68	48.00	0.00	56	404	28	0.40
d20c	66.00	0.17	66	845	0	0.48	65.00	0.00	66	64	0	0.09
d20d	89.00	0.55	92	2992	0	7.96	86.00	0.01	92	263	13	0.37
a40a	130.00	0.91	138	75169	250	25.54	130.00	0.00	138	4633	465	3.56
a40b	225.00	0.82	225	2954	0	1.42	220.00	0.00	225	101	0	0.06
a40c	349.59	1.16	367	412653	437	198.89	347.00	0.01	367	22803	1689	4.35
a40d	463.00	2.98	472	38026	9	50.74	457.03	0.01	472	3135	160	0.77
b40a	95.00	0.81	98	1629	0	3.73	92.00	0.02	98	90	0	0.14
b40b	144.64	1.54	162	1131747	1482	545.56	138.01	0.01	162	13764	911	16.51
b40c	165.86	1.90	167	5738	0	7.64	158.00	0.02	167	224	0	3.04
b40d	266.00	3.40	266	5472	0	2.96	262.01	0.02	266	181	0	0.81
c40a	56.17	1.20	57	2694	0	11.41	56.00	0.02	57	241	14	0.56
c40b	68.00	2.06	70	5544	0	15.42	68.00	0.01	70	140	0	25.48
c40c	99.25	3.03	103	18832	16	86.85	96.00	0.02	103	319	7	0.96
c40d	128.00	5.90	128	6358	0	6.86	128.00	0.02	128	129	0	0.22
d40a	43.50	1.34	48	4165	0	17.50	43.00	0.02	48	218	12	0.76
d40b	67.00	5.07	69	5031	0	45.25	67.00	0.03	69	3012	118	1.38
d40c	82.25	6.14	87	82742	81	284.93	81.00	0.04	87	13227	711	9.48
d40d	94.75	16.75	99	41530	1	335.59	93.00	0.04	99	169	0	0.34
a80a	177.00	8.57	180	18885	12	152.89	176.00	0.02	180	434	9	0.82
a80b	271.50	10.58	284	8608104	3407	20548.80	271.00	0.03	284	22516	560	54.65
a80c	376.86	26.19	379	103701	17	508.14	376.00	0.05	379	572	0	43.22
a80d	432.00	23.25	433	42330	4	329.70	421.00	0.04	433	2600	31	0.96
b80a	95.00	24.93	96	35180	18	285.33	95.00	0.04	96	192	0	41.01
b80b	161.50	31.35	163	93857	52	526.681	161.00	0.06	163	224	0	29.39
b80c	196.50	99.11	200	177107	70	976.50	192.00	0.07	200	11010	242	66.98
b80d	226.60	133.10	229	1744409	32	19180.20	226.00	0.10	229	2030586	58793	1213.70
c80a	68.00	22.39	73	2480024	2330	3494.11	65.00	0.10	73	63015	3864	92.53
c80b	89.20	55.27	90	27903	0	292.86	88.00	0.14	90	252	0	0.34
c80c	118.48	66.66	120	66417	18	832.50	115.00	0.13	120	9985	0	43.44
c80d	157.00	136.60	157	18400	0	50.98	157.00	0.15	157	370	0	0.36
d80a	51.00	22.39	51	8616	0	70.09	51.00	0.12	51	193	0	1.14
d80b	81.50	155.99	82	29669	0	376.10	80.00	0.15	82	347	0	4.13
d80c	109.14	150.91	111	4347387	715	30826.50	109.00	0.17	111	29537	657	43.84
d80d	121.17	244.79	122	286899	0	5865.03	121.00	0.27	122	6109	53	20.68

(*) No integer feasible solution exists for this instance.

4. Conclusion

This work introduces two formulations for the $(s - P - t)-$shortest-path problem. The model $(Q2)$ presents linear relaxed solutions that are very close to optimal ones. Nevertheless, exploring this feature by the MIP module of CPLEX showed to be very time consuming. In contrast, the model $(Q3)$, although obtaining in general weaker linear relaxed solutions than the model $(Q2)$, showed to be an efficient approach for solving this problem. It seems that exploring the use of compact extended formulations by linking dual and primal variables can constitute a new field of research for solving efficiently combinatorial optimization problems.

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